(j_4 is current density distribution due to expansion of the beam only in the last section). If there is a considerable reduction in the length of the last section, then due to the reduction in I_4 the value $j_4 \sim L_4^3$ and the current density in tails at the target falls as $L_4^{3/2}$.

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CALCULATION OF THE IRREGULAR INTERACTION OF SHOCK WAVES

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The problem is considered of irregular Mach interaction (reflection) of shock waves (SW). The flow structure is presented in Fig. 1a. At a certain point A there is formation of a reflected SW AB, a contact separation surface AL, and a strong SW AO with subsonic flow behind it. In the features of the mathematical definition of this problem it borders on the problem of spreading of a subsonic jet in an accompanying supersonic flow considered in [1]. In a nonviscous approximation Mach interaction of SW was considered in [2, 3] on the example of flow of an overexpanded jet in a flooded space. In the definition suggested in [1] it is possible to calculate Mach interaction taking account of gas viscosity.

1. In order to clarify the general features of Mach interaction we consider evolution of the interaction picture with an increase in the intensity of incident SW AA'. The intensity of this wave with a prescribed Mach number for the incident flow M_1, will be determined by the angle of its slope β_1. With regular interaction there is formation of a reflected SW whose slope angle is β_2 clearly depends on M_1 and β_1. With an increase in β_1 starting from some $\beta_1 = \beta_1^*$ two forms of interaction are theoretically possible: regular and Mach. For $\beta_1 > \beta_1^0$ only Mach interaction is possible; β_1^0 depends on M_1, or what is the same, on the ratio p_1/p_2, i.e., the pressures ahead of and behind an incident shock [4]. In spite of the fact that with $\beta_1 < \beta_2^0$ only regular reflection is observed by experiment, for the purposes of illustrating the effect of viscosity in pure form it is of interest theoretically with $\beta_1^* < \beta_1 < \beta_1^0$ to consider Mach interaction. With $\beta_1 > \beta_1^*$ the line of contact separation AL is directed towards the plane of symmetry, and the strong SW is curved. With departure from point A downwards over the flow along line AL the angle of slope of the contact surface tends towards zero and there is isoentropic compression of the outer supersonic flow. Mach number at line AL tends towards the value $M_3^* (\beta_1, M_1)$. With certain $\beta_1 = \beta_1^{**} > \beta_1^0$, $M_3^* = 1$, and the velocity equals sound velocity. For $\beta_1 < \beta_1^0$ flow in region ABL remains supersonic. The structure of flow in this case may be determined by only considering interaction between subsonic flow in jet OALO' and supersonic flow in region ABL. We call this form of irregular interaction isolated. With $\beta_1 > \beta_1^0$ only Mach interaction of SW is realized. For β_1 satisfying the condition $\beta_1^0 < \beta_2 < \beta_1^{**}$ flow along the whole of line AL with isoentropic compression remains supersonic. However, with interaction of compression waves formed in flowing around

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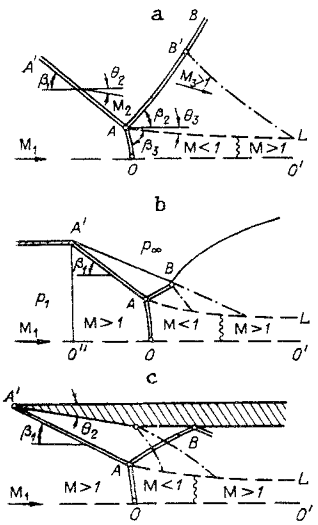

Fig. 1

AL, from reflected SW AB turning of the flow directly behind it by angle θ_2 in section B'B due to loss in the shock is only possible in subsonic flow. In spite of this under certain flow conditions in some finite region adjacent to point A the position of shock AB' may be worked out as before independent of conditions downward through the flow, although the possibility of physical realization of this flow is essentially determined by the conditions formed in the subsonic region of flow. This form of flow is also referred to as isolated. With $\beta_1 > \beta_1^{**}(M_1)$ flow along line AL only remains supersonic at some distance from point A. Therefore the region of supersonic flow AB'L (Fig. 1a) is localized in some finite vicinity of point A. In this case the flow picture behind the reflected SW depends markedly on conditions formed downward over the flow, and the statement of the problem of isolated Mach interaction becomes unclosed. In order to close the problem it is necessary to introduce additional disturbances. In practical problems they arise under the influence of conditions at the boundary of flow. For example, with flow behind a Mach disk in the overexpanded jets as an additional disturbance there is a rarefaction wave formed with reflection of jump AB from the boundary of the jet (Fig. 1b), and with flow in channels an additional disturbance arises with flow over the channel wall (Fig. 1c).

The change in pressure along the jet is governed by interaction conditions with supersonic flow in region ABL. Since the Mach interaction the angle of slope of the contact surface at point A is negative, and with departure downwards through the flow from this surface due to the effect of the plane of symmetry OO' it tends towards zero, then the contact surface should at least have areas of concavity. With supersonic flow past a concave contact surface there is an increase in pressure which causes retardation of a subsonic jet. This is not possible due to an increase in cross-sectional area of the jet bounded by surface AL and the requirement for mass conservation in the jet. Consequently, in the definition for a nonviscous gas the problem of isolated Mach interaction does not have a solution. This inconsistency is removed by considering the ejecting effect of supersonic flow which requires detailed consideration of viscous flow in the mixing layer replacing the contact separation in nonviscous gas. In this respect the problem of isolated interaction is similar to the problem of flow behind the base of a body in supersonic flow with presence of distributed subsonic injection which was considered in [1].

With unisolated interaction the change in pressure at line AL is determined not only by the shape of this line, but also the rarefaction wave impinging from outside under the action of which pressure decreases in a supersonic flow at a concave surface (Fig. 1). In this case a closed solution may be obtained also in an approximation for a nonviscous gas. The effect of viscosity is quantitative in nature. These heuristic considerations are reinforced below by the results of analyzing the mathematical definition of the problem.

2. We turn to a simple definition of the problem which considers the features enumerated above. We turn attention to the fact that although shock AO is curved, pressure along it changes little over a wide range of parameters. In view of this we shall consider that pressure is constant along the jet and in order to describe flow in it we use equations for a boundary layer:

$$\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=0,\ \rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}_*}\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y},$$
$$\rho u\frac{\partial H}{\partial x}+\rho v\frac{\partial H}{\partial y}=\frac{1}{\mathrm{Re}_*}\frac{\partial}{\partial y}\frac{\mu}{\mathrm{Pr}}\frac{\partial H}{\partial y}+\mu\left(1-\frac{1}{\mathrm{Pr}}\right)\left(\frac{\partial u}{\partial y}\right)^2.$$

Here uv_1, vv_1, $\rho\rho_1$, $p\rho_1 v_1^2$, hv_1^2, $\mu\mu_1$ are longitudinal and transverse components of the velocity vector, density, pressure, static enthalpy, and dynamic viscosity coefficient; $H = u^2/2 + h$; Pr and Re_* are Prandtl and Reynolds numbers. This set of equations should be solved with the following boundary conditions: at the nominal boundary of the viscous jet with $y = \delta(x)$ $u = u_\delta$, $H = H_\delta$, and at the line of symmetry with $y = 0$ $\partial u/\partial y = \partial H/\partial y = 0$, $v = 0$.

Pressure distribution p(x) is found as a result of solving the problem of interaction of flow in a viscous subsonic jet OALO' with an external nonviscous flow in region ABL which will be described by a complete set of Euler equations. The corresponding problem of viscous–nonviscous interaction is formulated in [1, 5]. It is shown that the pressure gradient may be connected with the change in effective displacement of the surface $y = \delta^*(x)$, with parameters in a viscous subsonic jet and at its boundary by the equation

$$\frac{dp_\delta}{dx}=\frac{\gamma p_\delta\frac{d\delta^*}{dx}+A_V}{\Delta},\tag{2.1}$$

where

$$A_V=\frac{\gamma-1}{\gamma}\frac{1}{\mathrm{Re}_*}\int_0^\delta\frac{1}{u}\left\{\frac{h}{u}\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y}-\frac{\partial}{\partial y}\frac{\mu}{\mathrm{Pr}}\frac{\partial h}{\partial y}-\mu\left(\frac{\partial u}{\partial y}\right)^2\right\}dy;$$
$$\Delta=\int_0^\delta\frac{\partial}{\partial y}\left(\frac{\mathrm{M}^2-1}{\mathrm{M}^2}\right)y\,dy-\frac{\mathrm{M}_\delta^2-1}{\mathrm{M}_\delta^2}\delta^*.$$

Equation (2.1) is considered as a differential equation with respect to unknown functions $p_\delta(x)$ and $\delta^*(x)$. The second equation emerges from boundary conditions for nonviscous supersonic flow [5]:

$$\frac{d\delta^*}{dx}\frac{dp_\delta}{dx}=\rho_\delta u_\delta^2\frac{d^2\delta^*}{dx^2}+\frac{\partial p}{\partial y}\left(1+\left(\frac{d\delta^*}{dx}\right)^2\right).\tag{2.2}$$

Here $\partial p/\partial y$ is derivative of pressure in the nonviscous part of the flow which is found numerically simultaneously with solution of the Euler equations. Equation (2.2) is an equation for pulse conservation written in a coordinate system connected with the flow line:

$$\frac{\partial p}{\partial n}+\rho u^2\frac{\partial\theta}{\partial s}=0.$$

Thus, we have a set of differential equations (2.1), (2.2) for determining unknown functions $p_\delta(x)$ and $\delta^*(x)$. For (2.1), (2.2) a Cauchy problem is formulated with starting data in section $x = 0$: from calculation of parameters at ternary point A and behind the normal shock $d\delta^*/dx$ and $p_\delta(0)$ are determined; as far as $\delta^*(0)$ is concerned, then the condition for finding it is formulated by proceeding from the following circumstances. In Eqs. (2.1) and (2.2), which should be solved together with the Euler and boundary layer equations, it is possible to reduce the value of Δ to zero with a certain value $x = x^*$. Here it is also necessary to reduce to zero simultaneously the numerator of Eq. (2.1). This is achieved by special selection of the value $\delta^*(0)$ equal to distance OA. A similar condition is used in order to determine the pressure in isobaric fragmented zones [6], i.e., the condition of closing the trace.

In the case of isolated Mach interaction if the reflection of disturbances which arise with flow past surface AL from a reflected SW AB is disregarded, flow in the nonviscous region may be described by solution for a simple wave with constant parameters along the first characteristic [7]. Then Eq. (2.2) may be rewritten in the form

$$\frac{1}{1+\left(\frac{d\delta^*}{dx}\right)^2}\frac{d^2\delta^*}{dx^2}=\frac{\sqrt{\mathrm{M}_\delta^2-1}}{\mathrm{M}_\delta^2}\frac{1}{\gamma p_\delta}\frac{dp_\delta}{dx}.\tag{2.3}$$

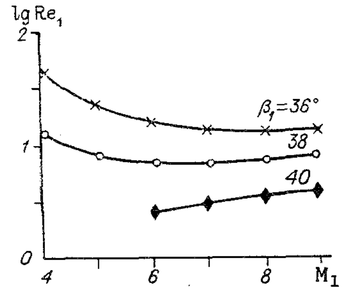

Fig. 2

If no consideration is given to the effect of viscosity and in order to describe flow in a subsonic jet a quasi-unidimensional model is adopted as in [2, 3], then from (2.1) it is easy to obtain

$$\left(1-\frac{1}{M_{\delta u}^2}\right)\frac{1}{\gamma p_\delta}\frac{dp_\delta}{dx}=-\frac{d\delta^*}{dx} \tag{2.4}$$

($M_{\delta u}$ is Mach number in the jet). From Eqs. (2.3) and (2.4) it is impossible to solve the problem of isolated interaction without considering viscosity. In fact, at point A $d\delta^*/dx = \tan\theta_3 < 0$, and with $M^2_{\delta u} < 1$, which occurs behind a strong SW, we obtain $dp_\delta/dx < 0$. In turn from (2.3) it emerges that $d^2\delta^*/dx^2 < 0$, by remaining negative with respect to the modulus it increases, which leads according to (2.2) to a further reduction in pressure and acceleration of subsonic flow, i.e., to an increase in $M_{\delta u}$. In the section where $M_{\delta u} = 1$ with finite $d\delta^*/dx$, $dp_\delta/dx \to -\infty$, which is physically impossible. Consideration of viscosity causes appearance of the term A_V in the numerator of (2.1), which it is possible with certain values of Re_* it is possible to obtain $dp_\delta/dx > 0$ with $d\delta^*/dx < 0$ and $M_{\delta u} < 1$. Under these conditions $d\delta^*/dx$ with respect to modulus decreases and tends towards zero. The mass balance condition in a subsonic jet as already noted may be fulfilled due to the ejecting effect of external flow.

With unisolated interaction it is necessary to consider Eq. (2.4) simultaneously with (2.2). With $M_{\delta u} < 1$ it follows from (2.4) that $(dp_\delta/dx)d\delta^*/dx > 0$. With impingement from outside on surface AL of a rarefaction wave of sufficiently high intensity ($\partial p/\partial y < 0$) it means that $d^2\delta^*/dx^2 > 0$. Under these conditions with negative dp_δ/dx and $d\delta^*/dx$ there is a reduction in modulus $d\delta^*/dx$ with an increase in x and cases are possible when $d\delta^*/dx \to 0$ with $M_{\delta u} \to 1$, i.e., the problem of unisolated Mach interaction for a nonviscous gas is resolvable in principle.

3. We consider the problem of isolated interaction. In it there is no characteristic linear dimension but there is a parameter with a dimension of length $\mu/\rho u$ which for this problem is a governing linear dimension. If as geometric characteristic dimension distance AO is selected, then in dimensionless variables $\delta^*(0) = 1$ and the unknown paramter becomes characteristic Reynolds number $Re_1 = \rho_1 u_1 AO/\mu_1$. It is necessary to select this so that in Eq. (2.1) with some x the numerator and the denominator simultaneously reduce to zero. The results of solving the problem are presented in Fig. 2 where the dependence of Re_1 on M_1 is shown for different β_1. It can be seen that Re_1 varies within the limits from 5 to 100, i.e., the distance is determined by the ratio $AO = 5-100\ \mu_1/\rho_1 u_1$, and under real conditions flow is negligibly small. The smallness of Re_1 for the problem of isolated interaction gives some basis for considering a laminar flow regime behind a Mach blade before the closing section.

4. We consider the case of isolated interaction on the specific example of flow of plane and axisymmetrical overexpanded jets. Jet parameters will be found with respect to M_1 and angle β_1 for a shock arising at the nozzle outlet. From them it is easy to determine the jet operating conditions. Linear dimensions are related to half the width of the jet at the nozzle outlet. Since with quite high Reynolds numbers, which occur for this problem, flow in the mixing layer is hydrodynamically unstable, we shall consider that flow in a subsonic jet is turbulent. As a turbulence model we use the simple Prandtl algebraic model [8] and the Sekundov differential turbulence model [9]. The height of the Mach blade $\bar{y}_d$ is unknown which is subject to determination in solving the interaction problem. Presented in Fig. 3 are the dependences of $\bar{y}_d$ on M_1 with different values of β_1. Solid and broken lines

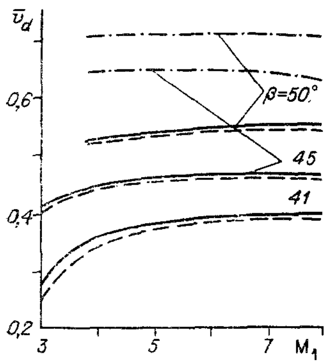

Fig. 3

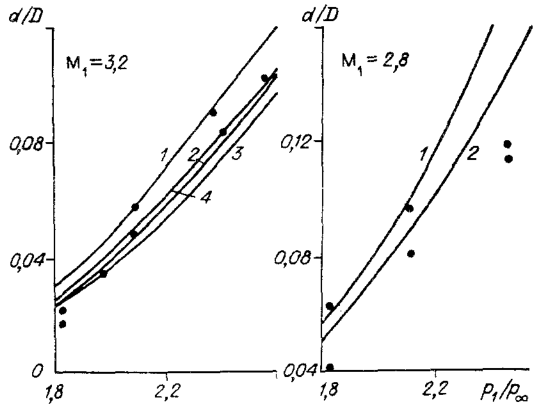

Fig. 4

are the results of calculations for plane flow in a jet in a nonviscous approximation and taking account of viscosity respectively, and perpendicular lines are calculated curves for axisymmetrical flow in a jet taking account of viscosity. All of the results provided were obtained using the simplified Prandtl equation for turbulent viscosity [8] with normal values of proportionality coefficient $\varkappa = 0.03$. Shown in Fig. 4 is the dependence of Mach disk diameter d/D (D is channel diameter) on the ratio of pressure at the outlet of an axisymmetrical nozzle p_1 to the pressure in the surroundings p_∞. Theoretical (curve 1) and experimental (points) values are taken from [3]. Curve 2 relates to calculation by the method suggested taking account of viscosity for the Prandtl algebraic turbulence model, and 3 and 4 for the Sekundov differential turbulence model [9] with a starting turbulence condition $\mu_{t0} = 0.002$ and 0.0002, respectively.

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